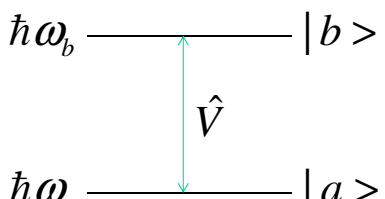
光量子エレクトロニクス特論

1. 電磁気学の復習
2. 量子力学の復習
3. 状態ベクトルとシュレディンガー方程式
4. 時間依存のシュレディンガー方程式
5. 密度行列法
6. 電子と光の相互作用-古典振動子モデル
7. 電子2準位系とブロッホ方程式
8. 電子2準位系におけるコヒーレント光非線形効果
9. 2原子分子のエネルギー準位
10. 固体のエネルギー準位
11. 光学遷移と次元性
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15. まとめ

2準位系のブロッホ方程式

$$\dot{\rho} = -\frac{i}{\hbar}[\hat{H}, \rho]$$

$$\hat{H} = \hat{H}_0 + \hat{V} + \hat{H}_{\text{relax}}$$



$$\hat{H}_0 = \begin{pmatrix} \hbar\omega_a & 0 \\ 0 & \hbar\omega_b \end{pmatrix} \qquad \hat{V} = \begin{pmatrix} 0 & V_{ab} \\ V_{ba} & 0 \end{pmatrix}$$

$$\dot{\rho}_D = -\frac{2i}{\hbar}(V_{ab}\rho_{ba} - \rho_{ab}V_{ba}) - \frac{1}{T_1}(\rho_D - \rho_D{}^{(0)})$$

$$\dot{\rho}_{ba} = -\frac{i}{\hbar}V_{ba}\rho_D - \left(\frac{1}{T_2} + i\omega_0\right)\rho_{ba}$$

T_1：縦(分布)緩和時間　　T_2: 横(位相)緩和時間

摂動展開

$$\rho = \rho^{(0)} + \rho^{(1)} + \rho^{(2)} + \rho^{(3)} + \cdots$$

摂動：電場 E の次数

$\rho^{(0)}$ ：熱平常状態　$\rho_{ba}{}^{(0)} = 0$

$$\dot{\rho}_D{}^{(n)} = -\frac{2i}{\hbar}(V_{ab}\rho_{ba}{}^{(n-1)} - \rho_{ab}{}^{(n-1)}V_{ba}) - \frac{1}{T_1}\rho_D{}^{(n)}$$

$$\dot{\rho}_{ba}{}^{(n)} = -\frac{i}{\hbar}V_{ba}\rho_D{}^{(n-1)} - \left(\frac{1}{T_2} + i\omega_0\right)\rho_{ba}{}^{(n)}$$

$$V_{ba} = -\mu_{ba}\hat{E}e^{-i\omega t}$$

$$\rho_{ba}{}^{(n)}(t) = \sigma_{ba}{}^{(n)}(t)e^{-i\omega t}$$

$$\dot{\rho}_{ba}{}^{(n)} = -\frac{i}{\hbar}V_{ba}\rho_D{}^{(n-1)} - \left(\frac{1}{T_2} + i\omega_0\right)\rho_{ba}{}^{(n)}$$

↓

$$\dot{\sigma}_{ba}{}^{(n)} = -\frac{i}{\hbar}V_{ba}\rho_D{}^{(n-1)}e^{i\omega t} - \left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)\sigma_{ba}{}^{(n)}$$

2ωの項を落とす(回転波近似)

$$\dot{\sigma}_{ba}{}^{(n)} = \frac{i\mu_{ba}}{\hbar}\hat{E}\rho_D{}^{(n-1)} - \left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)\sigma_{ba}{}^{(n)}$$

↓

$$\sigma_{ba}{}^{(n)}(t)$$

$$= \frac{i\mu_{ba}}{\hbar}\exp\left[-\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t\right]\int_{-\infty}^{t}\hat{E}(t')\rho_D{}^{(n-1)}\exp\left[\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t'\right]dt'$$

$$\dot{\rho}_D^{(n)} = -\frac{2i}{\hbar}(V_{ab}\rho_{ba}^{(n-1)} - \rho_{ab}^{(n-1)}V_{ba}) - \frac{1}{T_1}\rho_D^{(n)}$$

↓

$$\dot{\rho}_D^{(n)} = -\frac{2i}{\hbar}(-\mu_{ab}\hat{E}^*\sigma_{ba}^{(n-1)} + \sigma_{ab}^{(n-1)}\mu_{ba}\hat{E}) - \frac{1}{T_1}\rho_D^{(n)}$$

↓

$$\rho_D^{(n)}(t) = \frac{2i}{\hbar}\exp\left(-\frac{1}{T_1}t\right)\int_{-\infty}^{t}\left\{\mu_{ab}\hat{E}^*(t')\sigma_{ba}^{(n-1)} - \sigma_{ab}^{(n-1)}\mu_{ba}\hat{E}(t')\right\}\exp\left(\frac{1}{T_1}t'\right)dt'$$

*n*次の分極

$$P^{(n)} = N\mathrm{Tr}(\rho^{(n)}\mu) = N(\rho_{ab}^{(n)}\mu_{ba} + \rho_{ba}^{(n)}\mu_{ab}) = N(\rho_{ba}^{(n)*}\mu + \rho_{ba}^{(n)}\mu^*)$$
$$= N(\mu^*\sigma_{ba}^{(n)}e^{-i\omega t} + \mu\sigma_{ba}^{(n)*}e^{i\omega t}) = \hat{P}^{(n)}e^{-i\omega t} + \hat{P}^{(n)*}e^{i\omega t}$$

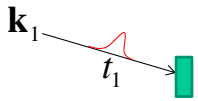
*n*次の分極

$$\hat{P}^{(n)} = N\mu_{ab}\sigma_{ba}^{(n)}$$

1次の分極

$$\hat{P}^{(1)} = N\mu_{ab}\sigma_{ba}^{(1)}$$
$$= \frac{iN\mu^2}{\hbar}\exp\left[-\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t\right]\int_{-\infty}^{t}\hat{E}(t')\rho_D^{(0)}\exp\left[\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t'\right]dt'$$

$$\hat{E}(t) = \theta_1\delta(t - t_1)e^{i\mathbf{k}_1\cdot\mathbf{r}}$$

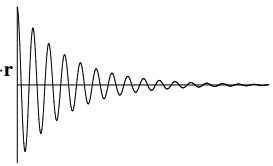


↓

$$\hat{P}^{(1)} = \begin{cases} \dfrac{iN\mu^2}{\hbar}\rho_D^{(0)}\theta_1\exp\left[-\left(\dfrac{1}{T_2} + i(\omega_0 - \omega)\right)(t - t_1)\right]e^{i\mathbf{k}_1\cdot\mathbf{r}} & t > t_1 \\ 0 & t < t_1 \end{cases}$$

1次の分極

$$\hat{P}^{(1)}(t) = \frac{iN\mu^2\rho_D^{(0)}}{\hbar}\exp\left[-\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)(t - t_1)\right]e^{i\mathbf{k}_1\cdot\mathbf{r}}$$



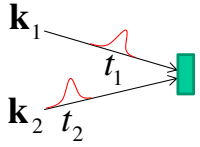
$\omega \simeq \omega_0$ のとき、この分極が作る電場の強度$I(t)$は

$$I(t) \propto |\hat{P}^{(1)}(t)|^2 = \frac{N^2\mu^4\rho_D^{(0)2}}{\hbar^2}\theta_1^2\exp\left[-\frac{2}{T_2}(t - t_1)\right], (t > t_1)$$

Free Induction Decay (自由誘導減衰)

3次の分極

$$\hat{P}^{(3)} = N\mu_{ab}\sigma_{ba}^{(3)}$$
$$\hat{E} = \tilde{E}_1e^{i\mathbf{k}_1\cdot\mathbf{r}} + \tilde{E}_2e^{i\mathbf{k}_2\cdot\mathbf{r}} \qquad \tilde{E}_j = \theta_j\delta(t - t_j)$$



$$\sigma_{ba}^{(3)}(t)$$
$$= \frac{i\mu_{ba}}{\hbar}\exp\left[-\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t\right]\int_{-\infty}^{t}\hat{E}(t')\rho_D^{(2)}(t')\exp\left[\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t'\right]dt'$$

$$\rho_D^{(2)}(t')$$
$$= \frac{2i}{\hbar}\exp\left(-\frac{1}{T_1}t'\right)\int_{-\infty}^{t'}\left\{\mu_{ab}\hat{E}^*(t'')\sigma_{ba}^{(1)}(t'') - \sigma_{ba}^{(1)*}(t'')\mu_{ba}\hat{E}(t'')\right\}\exp\left(\frac{1}{T_1}t''\right)dt''$$

$$\sigma_{ba}^{(1)}(t'')$$
$$= \frac{i\mu_{ba}}{\hbar}\exp\left[-\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t''\right]\int_{-\infty}^{t''}\hat{E}(t''')\rho_D^{(0)}\exp\left[\left(\frac{1}{T_2} + i(\omega_0 - \omega)\right)t'''\right]dt'''$$

電場の積に注目

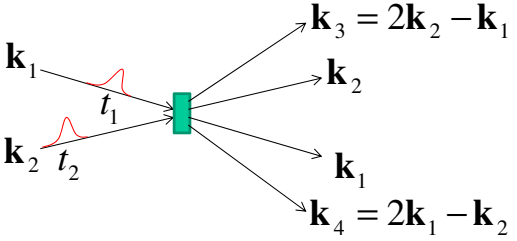
$$\hat{E}(t')\left\{\hat{E}^*(t'')\sigma_{ba}{}^{(1)}(t'')+\hat{E}(t'')\sigma_{ba}{}^{(1)*}(t'')\right\}$$

$$\rightarrow \hat{E}(t')\hat{E}^*(t'')\hat{E}(t''')+\hat{E}(t')\hat{E}(t'')\hat{E}^*(t''')$$

$$=\left\{\tilde{E}_1(t')e^{i\mathbf{k}_1\cdot\mathbf{r}}+\tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\right\}\left\{\tilde{E}_1{}^*(t'')e^{-i\mathbf{k}_1\cdot\mathbf{r}}+\tilde{E}_2{}^*(t'')e^{-i\mathbf{k}_2\cdot\mathbf{r}}\right\}\left\{\tilde{E}_1(t''')e^{i\mathbf{k}_1\cdot\mathbf{r}}+\tilde{E}_2(t''')e^{i\mathbf{k}_2\cdot\mathbf{r}}\right\}$$

$$+\left\{\tilde{E}_1(t')e^{i\mathbf{k}_1\cdot\mathbf{r}}+\tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\right\}\left\{\tilde{E}_1(t'')e^{i\mathbf{k}_1\cdot\mathbf{r}}+\tilde{E}_2(t'')e^{i\mathbf{k}_2\cdot\mathbf{r}}\right\}\left\{\tilde{E}_1{}^*(t''')e^{-i\mathbf{k}_1\cdot\mathbf{r}}+\tilde{E}_2{}^*(t''')e^{-i\mathbf{k}_2\cdot\mathbf{r}}\right\}$$

波数: $\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3=2\mathbf{k}_2-\mathbf{k}_1,\mathbf{k}_4=2\mathbf{k}_1-\mathbf{k}_2$



$\tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_1{}^*(t'')e^{-i\mathbf{k}_1\cdot\mathbf{r}}\tilde{E}_2(t''')e^{i\mathbf{k}_2\cdot\mathbf{r}}$,
$\tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_2(t'')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_1{}^*(t''')e^{-i\mathbf{k}_1\cdot\mathbf{r}}$

$\mathbf{k}_3=2\mathbf{k}_2-\mathbf{k}_1$ の成分

$$\tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_1{}^*(t'')e^{-i\mathbf{k}_1\cdot\mathbf{r}}\tilde{E}_2(t''')e^{i\mathbf{k}_2\cdot\mathbf{r}},\ \tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_2(t'')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_1{}^*(t''')e^{-i\mathbf{k}_1\cdot\mathbf{r}}$$

$t_1<t_2, t'>t''>t'''$ なので

$\tilde{E}_2(t')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_2(t'')e^{i\mathbf{k}_2\cdot\mathbf{r}}\tilde{E}_1{}^*(t''')e^{-i\mathbf{k}_1\cdot\mathbf{r}}$ だけが残る

$$\sigma_{ba}{}^{(3)}(t,\mathbf{k}_3)$$

$$=-2i\rho_D{}^{(0)}\left(\frac{\mu_{ba}}{\hbar}\right)^3\exp\left[-\left(\frac{1}{T_2}+i\Delta\omega\right)t+i\mathbf{k}_3\cdot\mathbf{r}\right]$$

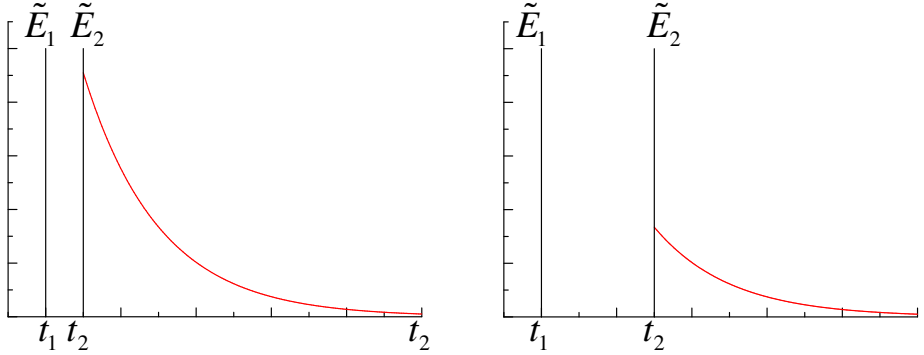
$$\times\int_{-\infty}^{t}dt'\int_{-\infty}^{t'}dt''\int_{-\infty}^{t''}dt'''\tilde{E}_2(t')\tilde{E}_2(t'')\tilde{E}_1{}^*(t''')\exp\left[\left(\frac{1}{T_2}-\frac{1}{T_1}\right)(t'-t'')+\frac{1}{T_2}t'''+i\Delta\omega(t'+t''-t''')\right]$$

$\Delta\omega=\omega_0-\omega$ $\tilde{E}_j=\theta_j\delta(t-t_j)$

$\Delta\omega=\omega_0-\omega\approx 0$ $\tilde{E}_j=\theta_j\delta(t-t_j)$

$$\sigma_{ba}{}^{(3)}(t,\mathbf{k}_3)$$

$$=\begin{cases}-2i\rho_D{}^{(0)}\left(\dfrac{\mu_{ba}}{\hbar}\right)^3\theta_1\theta_2{}^2\exp\left[-\dfrac{1}{T_2}(t-t_1)+i\mathbf{k}_3\cdot\mathbf{r}\right], & t>t_2\\ 0, & t<t_2\end{cases}$$



$$\hat{P}^{(3)}(t,\mathbf{k}_3)=N\mu_{ab}\sigma_{ba}{}^{(3)}(t,\mathbf{k}_3)$$

$$=\begin{cases}-2iN\mu_{ab}\rho_D{}^{(0)}\left(\dfrac{\mu_{ba}}{\hbar}\right)^3\theta_1\theta_2{}^2\exp\left[-\dfrac{1}{T_2}(t-t_1)+i\mathbf{k}_3\cdot\mathbf{r}\right], & t>t_2\\ 0, & t<t_2\end{cases}$$

この分極が作る電場の強度I(t)は

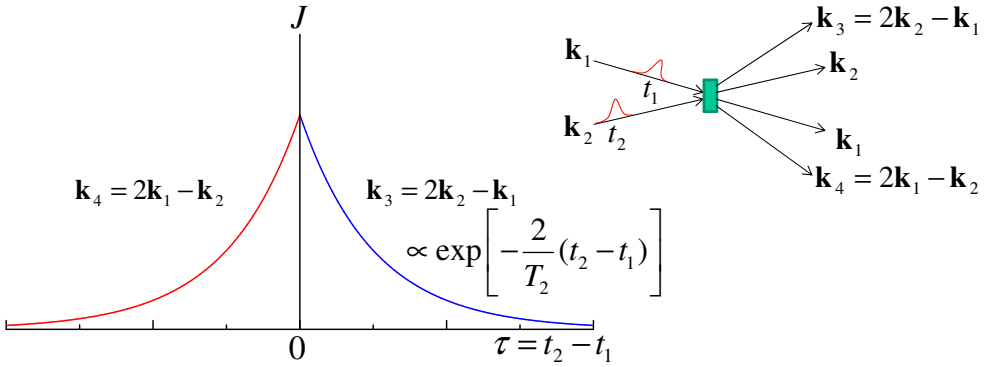
$$I(t,\mathbf{k}_3)\propto|\hat{P}^{(3)}(t)|^2=\frac{4N^2\mu^8}{\hbar^6}\rho_D{}^{(0)2}\theta_1{}^2\theta_2{}^4\exp\left[-\frac{2}{T_2}(t-t_1)\right],(t>t_2)$$

この強度$I(t)$時間積分$J(\tau=t_2-t_1)$は

$$J(\tau=t_2-t_1,\mathbf{k}_3=2\mathbf{k}_2-\mathbf{k}_1)=\int_{-\infty}^{\infty}I(t)dt=\int_{t_2}^{\infty}I(t)dt$$

$$=\frac{T_2N^2\mu^8}{2\hbar^6}\rho_D{}^{(0)2}\theta_1{}^2\theta_2{}^4\exp\left[-\frac{2}{T_2}(t_2-t_1)\right],(t_2-t_1>0)$$

縮退四光波混合法



2つのレーザー光パルスの時間間隔で積分強度が変わる
→ 検出器など測定器の時間応答は遅くても、
短い位相緩和時間の測定が行える。
→縮退四光波混合法(Degenerate Four Wave Mxing)

→フォトン・エコー(Photon Echo)